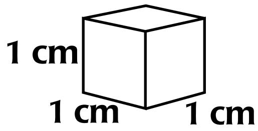
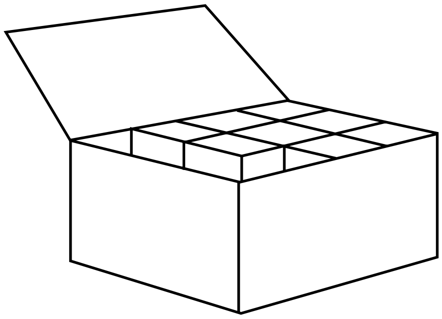
Activity

Volume and Surface Area

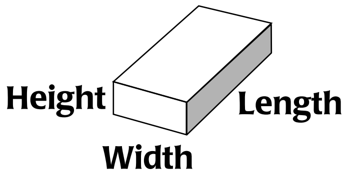
You have investigated the relationship between perimeter and area. In a similar way, you can look at the relationship between volume and surface area. This can be thought of as the relationship between the space inside a package and the area that can be covered by its wrapping.

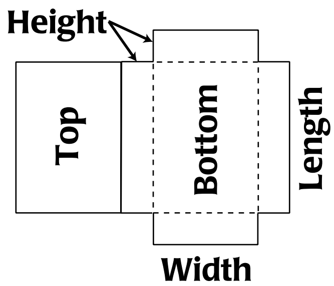




A rectangular box that will hold exactly 24 cubes is said to have a volume of 24 cubic centimeters, abbreviated as 24 cm^3.

21. Use centimeter cubes to find as many different-sized boxes as you can that will hold exactly 24 cubes. The dimensions of each box should be whole numbers. Find out how much cardboard would be needed to make each box (this is the surface area). Use a table like the one below to record your results.





Length (in cm)	Width (in cm)	Height (in cm)	Volume (in cm^3)	Surface Area (in cm^2)
			24	
			24	
			24	
			24	
			24	
			24	
			24	

Solutions and Samples
of student work

21. Tables will vary. Sample student response:

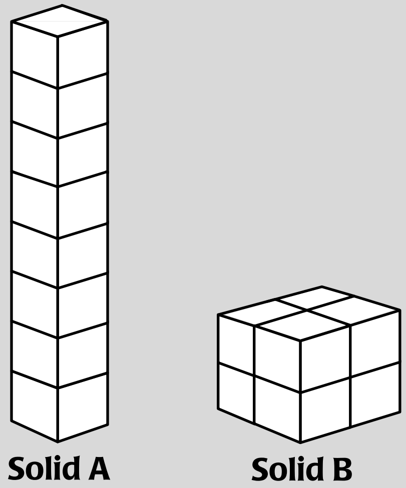
Length (in cm)	Width (in cm)	Height (in cm)	Volume (in cm^3)	Surface Area (in cm^2)
12	2	1	24	76
6	4	1	24	68
4	3	2	24	52
6	2	2	24	56
24	1	1	24	98
8	3	1	24	70

Hints and Comments

Materials centimeter cubes, optional (24 per pair or group of students)

Overview Students create different rectangular boxes, all with the same volume, and find the surface area of each.

About the Mathematics Area can be used to describe the surface area of a three-dimensional solid as well as a two-dimensional shape. This activity illustrates that volume and surface area are not directly proportional. For example, the two solids below each have a volume of 8 cubic units. Solid A has a surface area of 34 square units, while the surface area of Solid B is 24 square units.

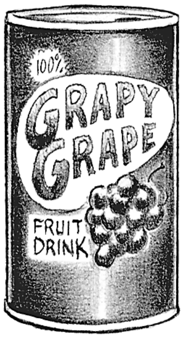


Planning Have students work in pairs or small groups on problem **21.** Discuss students' results in class.

Comments about the Problems

21. Encourage students to use the centimeter cubes. Making a table to systematically list the solutions can help students find all the possible different-sized boxes.

Ask students to think about the box with the smallest surface area and the one with the largest surface area. (The more a box resembles a cube, the smaller its surface area.)

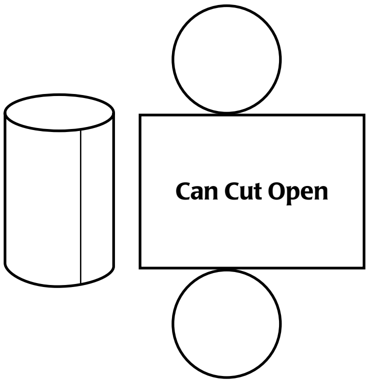


Fruit drinks come in cans of different sizes. Some cans are narrow and tall; others are wide and short. A juice can is made up of two circles and a rectangle. Cans that look completely different may contain the same amount of liquid or hold different amounts.

22. The type of fruit drink pictured on the left is also available in cans that are twice as high as the can you see in the picture.

a. How do the amounts of liquid that fit in the two cans compare?

b. What do you know about how the surface areas of the cans compare? Be prepared to explain your answer without making calculations.



23. Suppose one can has double the diameter of another can.

a. Do you think the amount of liquid that fits in the larger can will be double? Be prepared to explain your answer.

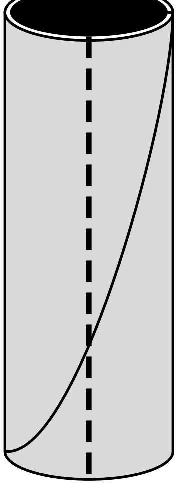
b. What can you tell about the surface area of the larger can compared to that of the original can?

Metro's Grocery sells juice in liter bottles. Many producers are switching to the metric system for measuring and bottling liquids. Any 10-centimeter cube will hold exactly 1 liter of liquid.

24. **a.** How many cubic centimeters are in one liter?

b. Describe the volume of 3 liters of juice in two different ways.

Collect cardboard tubes from inside several rolls of toilet paper.



25. **a.** What kind of shape do you get when you cut a tube along its spiral line? Make a drawing.

b. Cut another tube along a straight line, as shown in the picture on the left. Make a drawing of the resulting shape.

The two shapes you cut the tube into show that the area of a parallelogram is equal to the area of a rectangle with the same base and height. The shape of the parallelogram that is created by cutting the cylinder depends on the steepness of the cutting line. Still, no matter how steep you make the cutting line, the base, the height, and the total surface area will be the same.

22. **a.** Answers will vary. Sample student response:

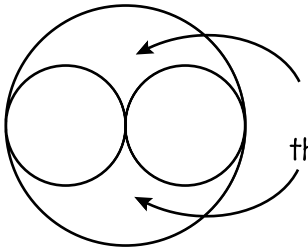
The taller can holds twice as much juice as the shorter can.

b. Answers will vary. Sample student response:

The surface area of the taller can is a little less than two times the surface area of the smaller can. The rectangular part of the surface is twice as big, but the ends of the can are the same size.

23. **a.** Answers will vary. Sample student response:

No. If you double the diameter, the can will hold more than twice as much juice. This drawing shows why:



b. Answers will vary. Sample student response:

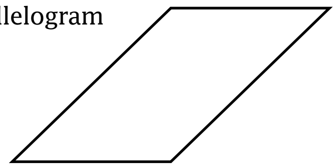
Doubling the diameter makes the surface area much larger. The area of the side doubles, but the areas of the top and bottom are four times larger.

24. **a.** 1,000 cm^3

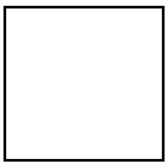
b. Answers will vary. Sample student response:

Three liters of juice is about $\frac{3}{4}$ gallon. This amount of juice would fill a box measuring 10 centimeters by 10 centimeters by 30 centimeters.

25. **a.** a parallelogram



b. a rectangle



Materials toilet paper rolls (two per group of students); scissors (one pair per group of students); cans (two per group of students); compasses, optional (one per pair or group of students)

Overview Students investigate the volumes and surface areas of cylinders using cans and toilet paper rolls. They explore the effect of doubling a cylinder's height and then doubling its diameter.

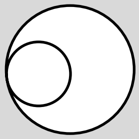
Planning Have students work in pairs or in small groups on problems **22–25.** Then discuss students' answers in class. You may want to use problem **24** for assessment.

Comments about the Problems

22–23. Students are not expected to accurately compute the surface area and volume. It is more important for them to reason as they explore relative sizes.

22. It may be helpful for students to physically place one can on top of another to see that the volume doubles.

23. If students have difficulty, suggest that they draw circles to help them visualize the two cylinders. They can trace the end of the can to create the smaller circle. Then they can use a compass to draw the larger circle.



24. **Informal Assessment** This problem assesses students' ability to understand the structure and use of standard systems of measurement, both metric and English.

25. Let students cut apart the cardboard tubes to transform them into parallelograms and rectangles. Students revisit strategies for determining the area of a parallelogram.