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Traditional academic disciplines in high schools often resemble silos. The grain stored in one never interacts with the grain stored in another. The discrete disciplines that frequently define the day of high school students resembles no known line of work other than teaching in high schools themselves. Once out of high school, students must quickly adapt to a world where the boundaries that defined their high school experience barely exist.

PREVIEW

The academic silos that house mathematics, science, and language skills must be broken down to better replicate how those disciplines mix in the real world.

Combining mathematics with writing promotes students' ability to analyze, compare facts, and synthesize information.

Written explanations enable teachers to better evaluate students' mathematical thinking.

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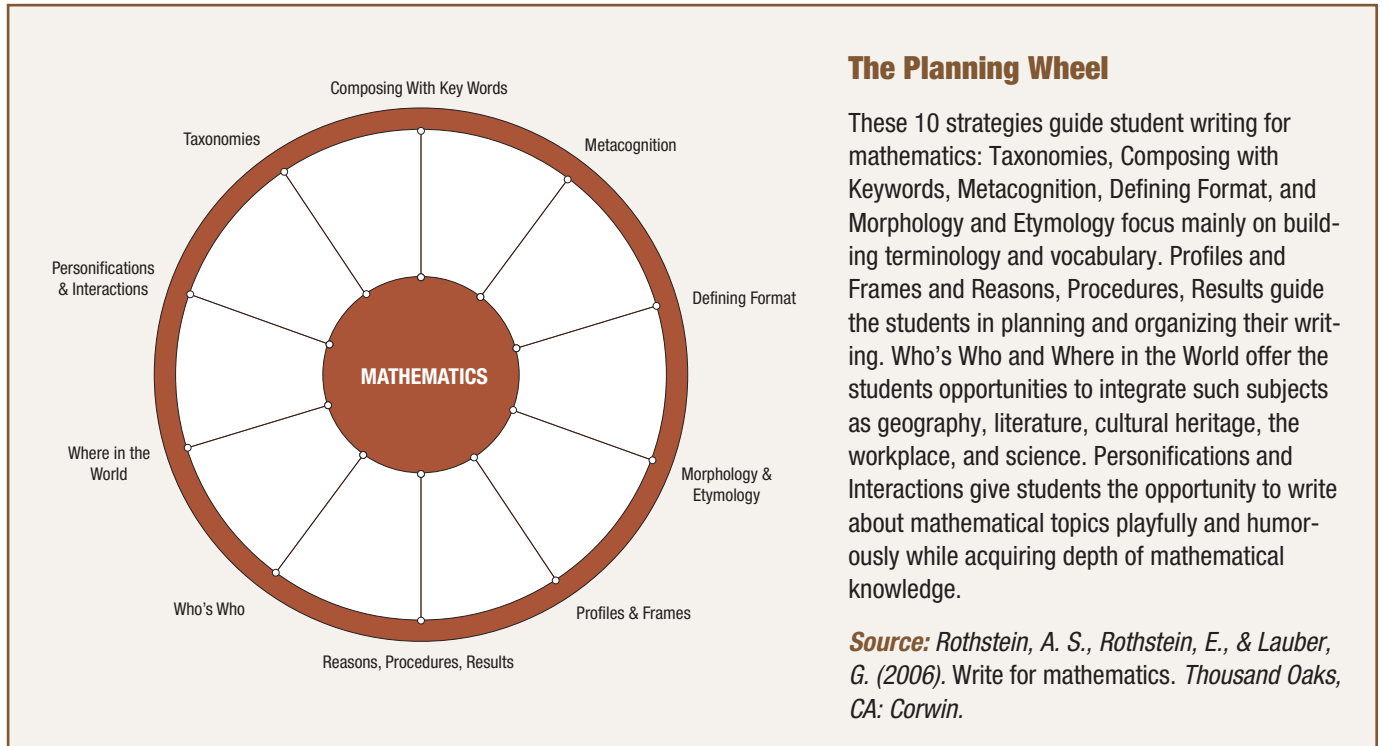
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They are the coauthors of Write for Mathematics (2006, Corwin).

Figure 1



Pink (2005) points out that the knowledge-based society of the United States is now morphing into a conceptual society. One of his key points is that academic knowledge alone will not protect workers from obsolescence and the impact of global competition. For educators, this economic evolution creates even greater urgency for integrated instruction and curriculum. Typically, teachers focusing on integrating curriculum gravitate toward those disciplines they consider kindred: English with social studies, mathematics with science. But this approach is far too limited to prepare students for a truly conceptual age.

A major cause of this societal change is the ease of communicating knowledge and ideas over the Internet. Among the concepts that Pink (2005) indicates will be needed in the future is the ability to turn data and information into a narrative. The direct consequence is that language skills, mathematics, and science learning must be intertwined.

Why Write in Mathematics?

Numerous mathematics educators have cited the benefits of linking writing with mathematics, pointing out that teachers who ask their students to write about mathematics are able to:

- Gain insight into their students' mathematical thinking

- Diagnose their students' misconceptions
- Assess students' study habits and attitudes
- Evaluate their own teaching techniques (National Council of Teachers of Mathematics [NCTM], 2000).

Other educators have focused on the importance of improving mathematical vocabulary, organizing ideas, solving problems, and clarifying mathematical concepts, all areas that combine numeracy with literacy. Miller (1991) points out that "improved mastery of mathematics concepts and skills is possible if students are asked to write about their understanding" (p. 520). Further benefits of combining mathematics with writing include promoting the ability to analyze, compare facts, and synthesize information (Kennedy, 1980; Russek, 2006). Studies by Freeman and Murphy (1992), Johnson (1983), and Sutton and Kruger (2002) indicate that this integration fosters greater student interest and higher student achievement levels in mathematics. Mathematics, after all, is a written language, and mathematicians write about mathematics.

Challenges

Teachers are often unprepared for the daunting task of integrating mathematics and writing. Mathematics teachers usually worry that teaching writing during mathematics class

takes time away from learning mathematics itself. They may also worry about correcting writing and spending time going over students' papers because they do not feel competent to examine student writing or they feel it is the role of the language arts teacher. Contrarily, language arts teachers spend little time teaching students genres of writing that rely largely on mathematics and are likely to feel ill at ease with mathematical content.

For the principal who sees the benefits of integrating curriculum, this "silo mentality" of subject separation presents a major challenge. Mathematics teachers often ask:

- How much time should be devoted to writing in mathematics?
- What strategies work best to integrate writing and mathematics?
- What are the genres of mathematical writing that students need to know?
- How can I be sure that the writing strategies I use in teaching mathematics are consistent with the writing demands in other subjects?

Mathematics teachers often see the expectation for integrating writing into the curriculum as an extra burden. Rarely can they recall their English teachers explaining how to write statements in algebra or geometry or their mathematics teachers helping them write a comparative essay about Roman numerals and Arabic numerals. Research by Gopen and Smith (1990), however, indicates that teachers can cover "the [mathematics] material" and "incorporate writing assignments...with significant success and without unduly burdensome effort" (p. 18). This research has also been corroborated by Burns (2004), Reeves (2002), and Countryman (1992).

Strategies for Mathematics Writing

A starting point for connecting the silos of mathematics and writing is to accept three key principles:

- Numeracy and literacy cannot be separated in the quest for mathematical achievement
- Writing and learning mathematics are a natural team
- Teachers need a wide range of writing strategies that are specifically suited to mathematics.

To support principals, teachers, and students, we researched the connections between writing and mathematics, then developed appropriate writing strategies that make writing and mathematics a unitary subject. Learning to write requires fluency plus organization. Simply stated, a writer must know the language of the subject or topic and the different

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types of organizational schema (also called *genres*) that are pertinent to mathematics. Writers must know the related vocabulary (e.g., operations, polynomials, axis, and perpendicular) to build their fluency and organization.

But knowing the vocabulary is only a starting point. To express their knowledge and application of these vocabulary terms, students must also know a variety of writing formats: definition, explanation, narration, and compare and contrast, among others. In mathematics, "developing fluency requires a balance and connection between conceptual understanding and computational proficiency" (NCTM, 2000, p. 35).

The concept of "fluency plus organization" in mathematics can be understood through the writings of Gardner (1993) who, in his work on multiple intelligences, defines *intelligence* as "the ability to solve problems or fashion products that are of consequence in a particular cultural setting or community" (p. 15). Gardner speaks of intelligences (plural) rather than intelligence (singular) to denote problem-solving ability in specific domains, such as verbal/linguistic, musical/ rhythmic, and logical/mathematical. He stresses the importance of focusing on both "the content of instruction and the means or medium for communicating that content" (p. 32).

To combine fluency (vocabulary and sentence composing) with organization (writing genres) we have focused on 10 strategies that guide students in writing for mathematics and have created a model called "the Planning Wheel" (figure 1). In this model, mathematics is in the center surrounded by instructional strategies that empower the students to express their learning and knowledge through writing. All of strategies in the Planning Wheel specifically target NCTM (2000) standards

Figure 2

Personifications and Interactions

Zero D. Cipher, the director of Indian/Arabic Mathematics, writes to Portia B. Romano, the director of Numbers in Rome, in response to her interest in the concept of zero. Such a letter as this can be used as an introduction to algebra. Providing the essential background knowledge about a concept before actually teaching it makes students confident and curious about what is to come.

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Dear Dr. Romano:

Thank you for your letter dated I/X/MC in which you ask for a more effective way to have Roman students do the mathematical operations of addition, subtraction, multiplication, and division. Yes, you are right. Your Roman numerals are great for putting on churches and other important edifices, but they have serious limitations when it comes to what we Indians and Arabs call “place value.” In your system, you have to do the computations mentally. On the other hand, we have figured out a regrouping system that uses zero and permits us to compute on paper and with relative ease.

Here’s a quick explanation of how this number we call *zero* works. For example, when zero is placed to the right of the numeral one or 1 it represents “no ones,” but signifies that the 1 is ten or in the tens place. That 10 is equivalent to your letter X. Now if we place two zeros to the right of the one as in 100, we now have ten of these tens or 10 of these 10s. From 100 or ten tens, we can go to 1000 or one hundred tens. So as you can see, with two symbols we can begin to express an infinite number of numbers. Of course, the only way you can understand infinity is to understand zero, but that is for another letter.

In your system, your students have to write these numbers as: X, C, M. This may be fine for buildings, but as you can see, there is no way that one can compute with your system.

I know this explanation must seem very complicated, but we will be delighted to send our finest mathematicians to Rome. They will get your students started on what we know will be a revolutionary change in the world of computation and a new branch of mathematics. We call this new branch algebra from an Arabic term *ilm aljebra*, which means to “repair or make better.”

Please do not hesitate to call on us, and in the meantime, we are including gratis our most recent books on place value and the power of zero.

Very sincerely,

Zero D. Cipher

Suggested Narratives on Mathematics

A History of Pi. P. Beckmann. 1993. New York: Barnes and Noble.

E=MC²: A Biography of the World's Most Famous Equation. D. Bodanis. 2000. New York: Berkley Press.

Five More Golden Rules: Knots, Codes, Chaos, and Other Great Theories of 20th Century Mathematics. J. L. Casti. 2001. New York: HarperCollins.

Innumeracy. J. A. Paulos. 2001. New York: Farrar, Straus, and Giroux.

The Mobius Strip. C. A. Pickover. 2006. New York: Thunder Mouth's Press.

Longitude: The True Story of a Lone Genius Who Solved the World's Greatest Scientific Problem of His Time. D. Sobel. 1995. New York: Walker and Company.

Zero: The Biography of a Dangerous Idea. C. Seife. 2000. New York: Penguin.

that focus on the importance of “communicating...mathematical thinking coherently and clearly” (p. 59).

These writing strategies help students apply their knowledge, gain proficiency in solving word problems, state reasons with proof, make connections between two mathematical ideas or concepts, explain their mathematical representations, and develop an interest in the big picture of mathematics and mathematical history. Further, they provide the springboard for complete curricular integration.

For example, an assignment that uses the Personifications and Interactions strategy requires two students to work together, with each one “personifying” a mathematical idea or concept. Each student writes a letter as this persona, giving explicit mathematical information to a related mathematical persona, including his or her “address” and “story” or narrative (figure 2). These letters carry within them a touch of humor which not only relieves tension and anxiety (and makes us laugh), but “represents one of the highest forms of human intelligence” (Pink, 2005, p. 190).

Breaking Down the Silos

The Personifications and Interactions example is only one of many ways that writing and mathematics can be integrated. It also corresponds to another one of the emerging demands of the conceptual age that Pink calls *play* (2005).

We know that humor is a powerful learning tool and this type of strategy brings whimsy to what might otherwise be viewed as a dull topic. By extension, integrating writing and mathematics increases student engagement while building student knowledge. Further, when the writing strategies are specifically aligned to word problems, mastering algorithms, or applying mathematical concepts, teachers naturally start integrating curriculum, with the added benefit of making mathematical literacy an enjoyable learning experience. **PL**

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